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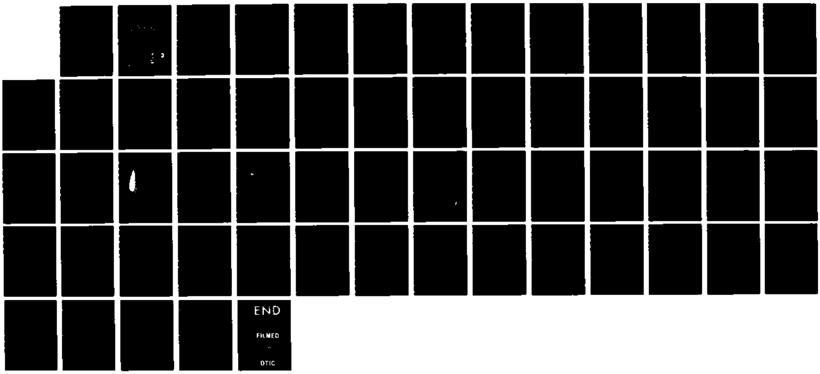

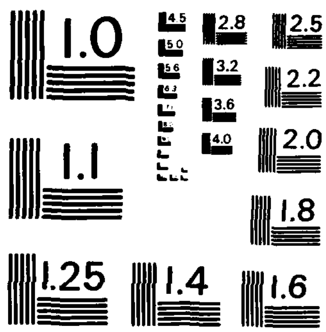

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TWO STABILITY RESULTS FOR DISCRETE ONE STEP AHEAD ADAPTIVE CONTROL

DOUGLAS SCOTT RHODE

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TWO STABILITY RESULTS FOR DISCRETE ONE STEP AHEAD
ADAPTIVE CONTROLLERS

BY

DOUGLAS SCOTT RHODE

B.S., University of Illinois, 1983

THESIS

Submitted in partial fulfillment of the requirements
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in the Graduate College of the
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CHAPTER 1

INTRODUCTION

Most control design methods require a complete characterization of the process to be controlled and its environment. Often in realistic settings, this knowledge is not directly available. One method to overcome this difficulty is to use an adaptive controller. Intuitively, an adaptive controller can adjust its behavior to various plants or disturbances. Although there are many types of adaptive systems, most can be decomposed into two loops as shown in Figure 1.1. The inner loop contains a plant and an ordinary regulator, while the outer loop adjusts the regulator to achieve some performance goal. A class of adaptive systems, model reference adaptive systems, incorporate a reference model in the outer loop. The response of the inner loop is adjusted so the plant output tracks the reference model output.

One of the simpler discrete model reference adaptive systems is the one step ahead adaptive controller presented by Goodwin (1984). The inner loop contains a plant which will be modelled as

$$A(q^{-1})y(k) = q^{-1}B(q^{-1})u(k) + w(k) \tag{1.1}$$

where $u(k)$ and $y(k)$ are the input and output, respectively, $A(q^{-1})$ and $B(q^{-1})$ are polynomials of unknown degree, and $\{w(k)\}$ is a sequence which may depend upon $\{u(k)\}$ and $\{y(k)\}$. Note that $\{w(k)\}$ may contain unmodeled dynamics as well as disturbances. The nominal plant is given when $w(k) = 0 \;\; \forall k$. The regulator is represented by

$$\phi(k)' \,\hat{\Theta}(k) = y_m(k+d) \tag{1.2}$$

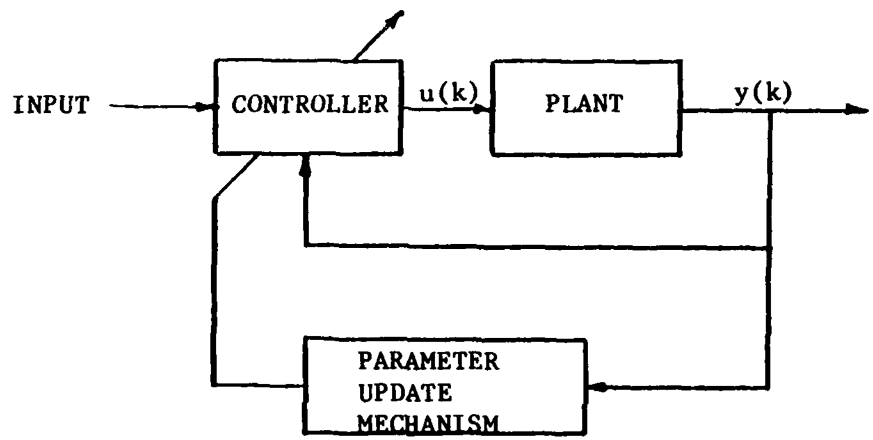

Figure 1.1. Generic adaptive system.

$$\phi(k)' \equiv (u(k),u(k-1),\ldots,u(k-m-d+1),y(k),y(k-1),\ldots,y(k-n+1))$$
$$\equiv (u(k) \;\vdots\; \phi_n(k)') \tag{1.3}$$

$$\hat{\Theta}(k)' = (\hat{\beta}_0(k),\ldots,\hat{\beta}_{m+d-1}(k),\hat{\alpha}_0(k),\ldots,\hat{\alpha}_{n-1}(k)' \equiv (\hat{\beta}_0(k) \;\vdots\; \hat{\Theta}\,(k)')$$

where $y_m(k+1)$ is the reference model output known one step in advance. The parameter update is done using a projection algorithm

$$\hat{\Theta}(k) = \hat{\Theta}(k-1) + \frac{\gamma(k)\phi(k-d)(y(k) - \hat{\Theta}(k-1)'\phi(k-d))}{1 + \gamma(k)\phi(k-d)'\phi(k-d)} \tag{1.4}$$

where $\gamma(k)$ is a scalar adaptation gain. With $\gamma(k)$ equals zero, the parameters are held constant and the system is linear time invariant as shown in Figure 1.2. The tracking transfer function is the transfer function from $y_m(k)$ to $y(k)$ with constant parameters. Although the stability properties of the system with constant parameters are well known, many issues are unresolved once parameter adaptation is allowed.

Goodwin, Ramadge and Caines (1980) proved the system signals are bounded and the tracking error tends to zero if three assumptions are made. First, they assumed the plant was disturbance-free, which in our case corresponds to $w(k) = 0, \forall k$. Second, the plant was assumed to be minimum phase. Without this assumption unstable unobservable modes could appear in the system if a controller pole cancelled an unstable zero. Finally, it was assumed that there existed a set of constant parameters for which the tracking transfer function was identically one. This condition is commonly referred to as matchability. Because most industrial processes violate one or more of these assumptions, more general results are needed.

Egardt (1983) extended the result of Goodwin, Ramadge and Caines (1980) to plants with bounded disturbances by projecting the parameters into

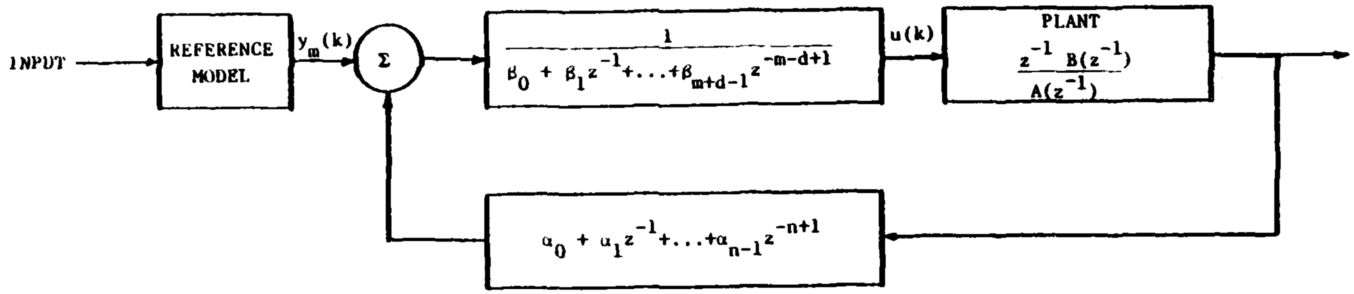

Figure 1.2. Adaptive system with fixed parameters.

a compact set to ensure their boundedness. Praly (1983) was able to extend Egardt's results to the case where

$$|w(k)| < \delta\, s(k) + w$$
$$s(k) = \mu\, s(k-1) + \max(\|\phi(k-d)\|, \Delta)$$
$$\Delta > 0 \quad , \quad 0 \leq \mu < 1 \tag{1.5}$$

when δ is sufficiently small by taking

$$\gamma(k) = \frac{1}{s(k)_0^2} \quad . \tag{1.6}$$

In Praly (1984), global stability is shown if there exists a constant parameter vector Θ^* for which the tracking transfer function has the following two properties:

1. The Nyquist plot of its causal part lies entirely within a circle of radius 1 and center 1. This is a <u>conicity condition</u>.

2. The noncausal part is sufficiently small.

The above results have not made any assumptions concerning the model reference output or the initial parameter values. If the initial regulator parameters result in a stable closed-loop system and the speed of convergence is not vital, then slow adaptation may be used. In slow adaptation the parameter update gain, $\gamma(k)$, is small, and if $\gamma(k)$ is sufficiently small, the system approximates a linear time invariant system.

As an extension of the work of Anderson and Johnson (1982), Kosut and Anderson (1984), Astrom (1984), Krause (1983), Anderson, Bitmead, Johnson and Kosut (1984), Riedle and Kokotovic (1984), Kokotovic, Riedle and Praly (1985),... on the stability of error models, it has been established in

Riedle, Praly and Kokotovic (1985), that if $\gamma(k)$ is small, the conicity condition mentioned previously can be relaxed to a signal dependent conicity condition. More explicity, the conicity condition can be replaced with the condition that the transfer function evaluated only at frequencies contained in the reference model output must lie in a circle of radius 1 and center 1.

In Chapter 2 a new matching condition is developed to ensure the local stability of the one step ahead adaptive controller for slow adaptation. It is assumed that there exists a constant parameter vector Θ^* such that the tracking transfer function is equal to one at the frequencies contained in a particular $\{y_m(k)\}$ called a test reference output. Then, it is shown that the adaptive system is robust to disturbances of the type described in Equation (1.5) and small bounded disturbances in the test reference output.

Advances in computer technology have lead to the widespread use of sampled data systems. Unfortunately, many of the stability results for discrete time adaptive controllers are not applicable to sampled data systems. The problem arises in that the equivalent discrete time model for a sampled continuous plant may have nonminimum phase zeros even if the plant is minimum phase. In fact, M'Saad et al. (1985) state that it is more of a rule than an exception for the discrete system to have nonminimum phase zeros.

As a result there is a need for discrete adaptive algorithms which can tolerate plants with nonminimum phase zeros. M'Saad et al. (1985) gives a comprehensive review of the literature dealing with such systems.

In most discrete reference model adaptive systems, the controller poles cancel the plant zeros to attain exact matching of the plant and reference outputs. If the plant has a nonminimum phase zero, the cancellation will lead to unobservable unstable system modes. Clearly the controller must

not cancel the unstable zero. One solution is to retain the nonminimum phase zero in the closed loop tracking transfer function. Johansson (1985) has established that the minimum *a priori* knowledge required is the unstable zero positions. Fortunately, Astrom, Hagander and Stemby (1984) have established the unstable zero positions for rapidly sampled continuous systems. They stated that for a continuous plant with n poles and m zeros:

1. The sampled-data representation of the system will have n-1 zeros and n poles; and, furthermore,
2. As the sampling period tends to zero, m zeros tend to 1 and are stable. The remaining n-m-1 zeros tend to known values which depend upon only the relative degree n-m, and if they are unstable, they are real and lie on the interval $(-\infty,-1]$.

Explicit use of this knowledge in an indirect scheme has been proposed by Clary and Franklin (1984).

In Chapter 3 a direct adaptive controller is proposed which assumes knowledge of the unstable zeros and retains them in the tracking transfer function. A global convergence analysis is presented for a disturbance-free plant, assuming matchability. The chapter finishes with a discussion of the strengths and weaknesses of the algorithm.

In Chapter 4 some concluding remarks and suggestions for further research are offered.

CHAPTER 2

LOCAL STABILITY OF A TUNED SOLUTION

This chapter develops an alternative matchability condition which will assure the local stability of a one step ahead adaptive controller for slow adaptation. First, the system and its equilibrium point are precisely defined. Then a nonlinear incremental model is formed. The incremental model is used to show that the linearized system is exponentially stable for the nominal plant. This result is then extended to derive a local stability result for the full system.

2.1. Preliminaries

In this section, an equivalent representation of the system is presented. With this representation, the tuning condition is defined and an incremental model is formed. This incremental model is the subject of the analysis presented in the following sections.

A more useful representation of the plant (1.3) will be the nonminimal state space representation defined as

$$X(k+1) = FX(k) + G_1u(k) + G_2w(k)$$

$$y(k) = h'X(k) \tag{2.1}$$

$$F = \begin{array}{c}\left[\begin{array}{cccccccc|cccccccc}
-a_1 & -a_2 & \cdots & -a_{\bar{n}} & 0 & \cdots & \cdots & 0 & b_2 & b_3 & \cdots & b_{\bar{m}} & 0 & \cdots & \cdots & 0 \\
1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & 1 & 0 & \cdots & \cdots & \vdots & \cdots & 0 & \vdots & & & & & & & \vdots \\
0 & \ddots & \ddots & \ddots & & \vdots & & \vdots & \vdots & & & & & & & \vdots \\
\vdots & & \ddots & \ddots & \ddots & \vdots & & \vdots & \vdots & & & & & & & \vdots \\
0 & \cdots & 0 & \cdots & 1 & 0 & 0 \cdots & 0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
\hline
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
\vdots & & & & & & & \vdots & 1 & 0 & & & & & & 0 \\
\vdots & & & & & & & \vdots & 0 & \ddots & \ddots & & & & & \vdots \\
\vdots & & & & & & & \vdots & \vdots & \ddots & \ddots & \ddots & & & & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & \cdots & 0 & 1 & 0 & \cdots & 0
\end{array}\right] \begin{array}{l} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\\1\end{array}}\right\} n^* \\ \left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\end{array}}\right\} m^* \end{array} \\
\longleftarrow n^* \longrightarrow \qquad \longleftarrow m^* \longrightarrow
\end{array}$$

$$G_1' = [\underbrace{b_1, 0, \ldots, 0}_{n^*} \mid \underbrace{1, 0, \ldots, 0}_{m^*}]$$

$$h' = G_2' = [1, 0, \ldots, 0]$$

$$X(k) = [y(k), \ldots, y(k-n^*+1), u(k-1), \ldots, u(k-m^*)]'$$

where a_i's and b_i's are the coefficients of $A(q^{-1})$, $B(q^{-1})$, respectively, and n^* and m^* are large enough for $X(k)$ to contain all the elements of $\phi(k-d)$. Then there exists a constant matrix H composed of ones and zeros such that

$$\phi(k-d) = HX(k) \quad . \tag{2.2}$$

Similarly, there exists a matrix J such that

$$\phi_r(k) = JX(k) \quad . \tag{2.3}$$

If Equations (2.2) and (2.3) are substituted into Equation (1.2), the control u(k) can be written explicitly as

$$u(k) = \frac{1}{\hat{\beta}(k)} [- \hat{\theta}_r(k)' J X(k) + y_m(k+d)] \quad . \tag{2.4}$$

For slow adaptation, $\gamma(k)$ is set to a small constant denoted by ε. It follows that the complete closed-loop adaptive system can be written as

$$X(k+1) = [F - \frac{G_1 \hat{\theta}_r(k)' J}{\hat{\beta}(k)}] X(k) + \frac{G_1}{\hat{\beta}(k)} y_m(k+d) + G_2 w(k)$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{\varepsilon H X(k) X(k)' [h - H\theta'(k-1)]}{1 + \varepsilon X(k)' H' H X(k)} \quad . \tag{2.5}$$

This system is highly nonlinear. In order to facilitate its study we now introduce the notion of a tuned solution. This will allow us to derive a more practical equivalent representation.

Definition: A test reference output is an N-periodic sequence $\{y_m^*(k)\}$ for which there exists an N-periodic sequence $\{X^*(k)\}$ and constant θ^* with $\beta^* \neq 0$ satisfying

i) $$X^*(k+1) = (F - \frac{G_1 \theta_r^{*\prime} J}{\beta^*}) X^*(k) + \frac{G_1}{\beta^*} y_m^*(k+d) \quad , \forall k \tag{2.6}$$

$$X^*(k)'(h - H'\theta^*) = 0 \quad . \tag{2.7}$$

ii) All eigenvalues of

$$(F - \frac{G_1 \theta_r^{*\prime} J}{\beta^*}) \tag{2.8}$$

are strictly inside the circle.

Equation (2.7) is called the tuning condition. In the appendix we show that this condition is generically satisfied if $y_m^*(k)$ does not contain more than $(m+d+n/2)$ frequencies. $(\theta^*, X^*(k))$ is called the tuned solution of the adaptive system.

The tuned solution represents an equilibrium of the adaptive system with $y_m^*(k+d)$ as the input. For notational convenience let

$$\phi^*(k-d) = HX^*(k) \tag{2.9}$$

$$\phi_r^*(k) = JX^*(k) \quad . \tag{2.10}$$

To continue with the analysis, the existence of an equilibrium must be assumed.

<u>Assumption A1</u>: For the nominal plant there exists a test reference output $\{y_m^*(k)\}$.

<u>Nonlinear Incremental Model</u>: The analysis is now focussed upon the behavior of system (2.5) around the tuned solution. For this purpose, define the incremental variables $x(k)$ and $\Theta(k)$ as

$$x(k) = X(k) - X^*(k) \tag{2.11}$$

$$\Theta(k) = \hat{\Theta}(k) - \Theta^* \quad . \tag{2.12}$$

The actual output reference $y_m(k)$ is obtained from the test output reference $y_m^*(k)$ by

$$y_m(k) = y_m^*(k) + v(k) \quad , \tag{2.13}$$

where $\{v(k)\}$ is a sequence satisfying

$$\sup_k |v(k)| \leq v \quad . \tag{2.14}$$

Substituting (2.11)-(2.14) into (2.5) the following equivalent incremental form is obtained:

$$\begin{bmatrix} x(k+1) \\ \Theta(k) \end{bmatrix} = M(k,\varepsilon)\begin{bmatrix} x(k) \\ \Theta(k-1) \end{bmatrix} + G\begin{bmatrix} v(k+d) \\ w(k) \end{bmatrix} + \begin{bmatrix} R_x(\Theta(k),x(k),v(k+d),k) \\ \varepsilon R_\Theta(\Theta(k-1),x(k),k) \end{bmatrix}, \quad (2.15)$$

where:

$$M(k,\varepsilon) = \begin{bmatrix} I & 0 \\ 0 & I+\varepsilon\phi^*(k-d)\phi^*(k-d)' \end{bmatrix}^{-1}\begin{bmatrix} F-G_1\frac{\theta_r^{*}J}{\beta^*} & -\frac{G_1}{\beta^*}\phi^*(k)' \\ \varepsilon\phi^*(k-d)(h'-\theta^{*\prime}H) & I \end{bmatrix}$$

$$G = \begin{bmatrix} \frac{G_1}{\beta^*} & G_2 \\ 0 & 0 \end{bmatrix}$$

$$R_x(\Theta(k),x(k),v(k+d)k) = \frac{G_1}{\beta^*(\beta^*+\beta(k))}[(\beta^*\Theta_r(k) - \beta(k)\Theta_r^*)'(\frac{\beta(k)}{\beta^*}\phi_r^*(k) - Jx(k))$$

$$- \beta(k)(v(k+d) - \frac{\beta(k)}{\beta^*}y_m^*(k+d))]$$

$$R_\Theta(\Theta(k-1),x(k),k) = \frac{H}{1+\varepsilon X(k)'H'HX(k)}\Big[[(I - \frac{\varepsilon(X(k)+X^*(k))'H'H}{1+\varepsilon X^*(k)'H'HX^*(k)})x(k)$$

$$\cdot((\Theta^{*\prime}H-h')x(k) + \phi^*(k-d)'\Theta(k-1))\Big] + X(k)x(k)'H'\Theta(k-1))].$$

The remainder of this chapter is devoted to proving the local stability of (2.15) around the origin. The incremental model is equivalent to the one step ahead adaptive controller of Equations (2.5). Two types of disturbances are considered.

Reference output perturbations are characterized by the bounded sequence v(k), and deviations of the plant from nominal are represented by w(k). The following assumption will allow the analysis to encompass a wide class of unmodeled effects.

Assumption A2: The sequence $w(k)$ satisfies

$$|w(k)| \leq \delta(s(k) + w) \quad , \quad \delta > 0 \quad , \quad w > 0$$

where

$$s(k) = \mu s(k-1) + \|X(k)\| \quad , \quad 0 < \mu < 1 \quad .$$

2.2. Exponential Stability of the Linear System

As a step towards the local analysis of (2.15), we consider the following linear time varying system:

$$\begin{bmatrix} x(k+1) \\ \Theta(k) \end{bmatrix} = M(k,\varepsilon) \begin{bmatrix} x(k) \\ \Theta(k-1) \end{bmatrix} \quad . \tag{2.16}$$

In the following theorem, established in Riedle *et al.* (1985), we take advantage of the structure of the matrix $M(k,\varepsilon)$ and the presence of the small parameter ε (the adaptation gain).

Let $v_o(k)$ be the unique N-periodic output of the following system:

$$L_o(k+1) = (F - \frac{G_1 \Theta_r^{*\prime} J}{\beta^*}) L_o(k) - \frac{G_1}{\beta^*} \phi^*(k)'$$

$$v_o(k) = -(h' - \Theta_*' H) L_o(k) + \phi^*(k-d)' \quad .$$

Theorem 1: Under Assumption A1, if the real parts of the eigenvalues of

$$\sum_{k=0}^{N-1} \phi^*(k-d) v_o(k)$$

are strictly positive, then there exists an $\varepsilon_o > 0$ such that for any ε, $0 < \varepsilon \leq \varepsilon_o$ there exist a $c(\varepsilon)$ and $\rho(\varepsilon)$ such that

$$\left\| \prod_{j=i+1}^{k} M(j) \right\| < c \, \rho^{k-i} \quad .$$

Proof: For the sake of completeness we give an outline of this proof. The essence of the proof is to rewrite system (2.16) in the form:

$$\begin{bmatrix} z(k+1) \\ \Theta(k) \end{bmatrix} = \bar{M}(k,\varepsilon) \begin{bmatrix} z(k) \\ \Theta(k-1) \end{bmatrix}$$

by the change of variables

$$z(k) = x(k) - L(k,\varepsilon)\Theta(k-1) \qquad ,$$

where $\bar{M}(k,\varepsilon)$ is a lower triangular matrix. The existence of the sequence of matrices $\{L(k,\varepsilon)\}$ which allows this transformation is provided by the following lemma:

Lemma: Under the assumptions of the Theorem, there exists an $\varepsilon_1 > 0$ such that for any ε, $0 \le \varepsilon \le \varepsilon_1$, the sequence $\{L(k,\varepsilon)\}$ exists with the following properties:

i) $L(k,\varepsilon)$ is K-periodic in k;

ii) $L(k,\varepsilon) = L_o(k) + \varepsilon L_1(k,\varepsilon)$ where $L_1(k,\varepsilon)$ is K-periodic and bounded;

iii) The matrix $\bar{M}(k,\varepsilon)$, defined above, is K-periodic and can be written as

$$\bar{M}(k,\varepsilon) = \begin{pmatrix} M_{11}(k,\varepsilon) & 0 \\ M_{21}(k,\varepsilon) & M_{22}(k,\varepsilon) \end{pmatrix}$$

and there exists $c_{11}(\varepsilon)$, $\rho_{11}(\varepsilon)$ such that:

$$\left| \prod_{j=i+1}^{k} M_{11}(j,\varepsilon) \right| < c_{11}\, \rho_{11}^{k-1} \qquad .$$

Proof: We write the nonlinear recursive equation satisfied by $L(k,\varepsilon)$. Then the existence of a unique K-periodic solution is shown using the contraction mapping theorem.

As a consequence of this lemma, the proof of the theorem is reduced to the study of $M_{22}(k,\varepsilon)$. It can be shown that

$$M_{22}(k,\varepsilon) = I - \varepsilon\phi^*(k-d)v_o(k)' + \varepsilon^2\bar{M}_{22}(k,\varepsilon)$$

where $M_{22}(k,\varepsilon)$ is bounded. The conclusion follows easily by noticing that

$$\prod_{k=0}^{N-1} \bar{M}_{22}(k) = I - \varepsilon \sum_{k=0}^{N-1} \phi^*(k-d)v_o(k)' + \varepsilon^2\bar{M} \quad .$$

This implies for some constant $\bar{m}$:

$$\left\| \prod_{k=0}^{N-1} \bar{M}_{22}(k) \right\| \leq 1 - \varepsilon\alpha + \varepsilon^2\bar{m} < 1$$

where the last inequality is obtained for ε small enough and

$$\mathrm{Re}\ \lambda_i\left(\sum_{k=0}^{N-1} \phi^*(k-d)v_o(k)'\right) > \alpha \qquad \forall i \quad .$$

This also shows that a meaningful approximation for ρ (as ε tends to zero) is

$$\rho = 1 - \varepsilon \min_i \mathrm{Re}\ \lambda_i\left(\sum_{k=0}^{N-1} \phi^*(k-d)v_o(k)'\right) \quad . \qquad \square$$

To complete our analysis of the linear system (3.1) we need to show that the assumption of Theorem 1 is satisfied. This is done with the following property which is proved in the appendix.

<u>Property 1</u>: Under Assumption A1, we have

$$v_o(k) = \phi^*(k-d) \quad , \quad \forall k \quad .$$

It follows that Theorem 1 holds provided we make an assumption on $\phi^*(k-d)$.

Assumption A3: The test reference output $y_m^*(k)$ is such that the corresponding tuned solution satisfies

$$\sum_{k=0}^{N-1} \phi^*(k)\phi^*(k)' > \alpha I \qquad \alpha > 0 \qquad .$$

This is the classical persistence of excitation assumption. In the appendix we show this condition is equivalent to the uniqueness of the tuned solution.

Under this assumption and for slow enough adaptation, the linear system (2.16) is exponentially stable.

2.3. Local Stability of the Adaptive System

As a well-known consequence of the uniform asymptotic stability of the linear system we have the following robustness result:

Theorem 2: Under Assumptions A1, A2 and A3 there exists an ε_o such that for any ε, $0 < \varepsilon \leq \varepsilon_o$ there exist $N_o(\varepsilon)$, $v_o(\varepsilon)$ and $\delta_o(\varepsilon)$ such that if $|X(0) - X^*(0)| + |\hat{\Theta}(0)-\Theta^*| < N_o(\varepsilon)$, then for any k, $v \leq v_o(\varepsilon)$ and $\delta \leq \delta_o(\varepsilon)$ we have $|X(k) - X^*(k)| + |\hat{\Theta}(k) - \Theta^*| \leq N$ for some constant N.

Proof: This type of result is known in a very general continuous-time context (for example see Theorem 5.2, Chapter X of Hale (1980)). Here we present a proof adapted to our particular problem in discrete time. Let Y(k+1) be the complete state of system (2.15). By the variation of constants formula, we have

$$Y(k+1) = \prod_{i=0}^{k} M(i)Y(0) + \sum_{i=0}^{k} \prod_{j=i+1}^{k} M(i) \left[G\begin{pmatrix} v(i+d) \\ \\ w(i) \end{pmatrix} + \begin{pmatrix} R_x(\Theta(i),x(i),v(i+d),i \\ \\ \varepsilon R_\Theta(\Theta(i-1),x(i),i) \end{pmatrix} \right] \quad . \tag{2.17}$$

We define

$$N(k) = \| Y(k) \| \quad .$$

Let ε, c, ρ be given by Theorem 1, then we can write with simplified notation:

$$N(k+1) \le C[\rho^k N(0) + \sum_{i=0}^{k} \rho^{k-i}[\|G\|(|v(i+d)| + |w(i)|) + \|R_x(i)\| + \varepsilon\|R_\Theta(i)\|] \quad .$$

The proof proceeds by induction. We assume,

$$N(i) \le N \quad \forall i \quad 0 \le i \le k \quad .$$

The objective is to show this implies

$$N(k+1) \le N$$

with an appropriate choice of N.

<u>Bound on w(i)</u>: By Assumption A2 ;

$$|w(i)| \le \delta(s(i) + w)$$

$$s(i) = \mu^i s(0) + \sum_{j=0}^{i-1} \mu^{i-j-1} \|X(j)\| \quad , \quad 0 \le \mu \le 1 \quad .$$

We also have

$$|X(j)| \leq \sup_k |X^*(k)| + |x(j)|$$

$$\leq \sup_k |X^*(k)| + N(j) \quad .$$

Therefore, with the induction assumption,

$$s(i) \leq s_o + \frac{\sup_k |X^*(k)| + N}{1-\mu} \quad .$$

This implies

$$|w(i)| \leq \frac{\delta}{1-\mu}(N + w^*) \quad , \quad 0 \leq i \leq k$$

with

$$w^* = \sup_j |X^*(j)| + (w + s_o)(1-\mu) \quad .$$

<u>Bound on R_x, R_Θ:</u> Assuming that $|\beta(k)| < \frac{|\beta^*|}{2}$ and using the Schwartz and triangle inequalities, it is easy to derive from the expressions for R_x and R_Θ the following inequalties:

$$|R_x(i)| \leq vN(i)P_3(N(i)) + N^2(i)P_4(N(i))$$

$$|R_\Theta(i)| \leq N^2(i)P_1(N(i)) \quad .$$

Where P_1, P_3 and P_4 are polynomials with positive coefficients of degree 1, 3 and 4, respectively. These polynomials are increasing functions of N(k) ($N(k) > 0$). For any $\bar{N} \geq N$ we conclude by the induction assumption:

$$P_j(N(i)) \leq P_j(\bar{N}) \;, \quad j=1,3,4 \quad , \quad 0 \leq i \leq k \quad .$$

It follows that a nondecreasing function, $K(\bar{N})$, can be found such that, if $|\beta(k)| \leq |\beta^*/2|$ then,

$$|R_x(i)| + \varepsilon |R_\theta(i)| \leq K(\bar{N})(vN + N^2) \quad , \quad 0 \leq i \leq k \quad , \quad N \leq \bar{N} \quad .$$

Substituting these bounds into Equation (4.1) gives

$$N(k+1) \leq C[(\rho N_o + \frac{|G|}{1-\rho}(v_o + \frac{\delta_o w^*}{1-\mu}) + (\frac{|G|\delta_o}{(1-\rho)(1-\mu)} + \frac{K(\bar{N})v_o}{(1-\rho)})N + \frac{K(\bar{N})N^2}{(1-\rho)}] \quad .$$

Therefore, $N(k+1)$ will be smaller than N if

$$C[(\rho N_o + \frac{|G|}{1-\rho}(v_o + \frac{\delta_o w^*}{1-\mu}) + (\frac{|G|\delta_o}{(1-\rho)(1-\mu)} + \frac{K(\bar{N})v_o}{1-\rho})N + \frac{K(N)N^2}{1-\rho}] - N \leq 0 \quad .$$

This inequality has a solution in N iff the following discriminant is nonnegative,

$$(-1 + \frac{C|G|\delta_o}{(1-\rho)(1-\mu)} + \frac{CK(\bar{N})v_o}{1-\rho})^2 - \frac{4K(N)C^2}{1-\rho}(N_o + \frac{|G|}{1-\rho}(v_o + \frac{\delta_o w^*}{1-\mu}) < 0 \quad .$$

Clearly, if v_o, δ_o, N_o are sufficiently small the inequality will be satisfied. To ensure $N < \bar{N}$ and $|\beta(k)| < \frac{|\beta^*|}{2}$ (with $|\beta(k)| < N$), note that $K(\bar{N})$ is an increasing function of $\bar{N}$, and as $K(\bar{N})$ increases, N will decrease.

2.4. Simulations

Through the use of several examples, this section demonstrates a variety of disturbances which satisfy Assumption A2. These disturbances are applied to a nominal system and the system is simulated.

<u>Nominal System</u>: For the ease of analysis a second order plant with delay 1 was chosen as nominal. This plant is represented by

$$(1 + a_1 q^{-1} + a_2 q^{-2}) y(k) = q^{-1}(b_1 + b_2 q^{-1}) u(k) \quad .$$

With $m = 0$ and $n = 1$ from Equation 2.4 the control is written as

$$u(k) = \frac{y_m(k+d) - \hat{\alpha}(k) y(k)}{\hat{\beta}(k)} \quad . \tag{2.18}$$

For constant $\hat{\beta}$ and $\hat{\alpha}$, the closed-loop tracking transfer function is

$$T_y(z^{-1}) = \frac{\frac{b_1}{\hat{\beta}} + \frac{b_2}{\hat{\beta}} z^{-1}}{1 + (a_1 + b_1 \frac{\hat{\alpha}}{\hat{\beta}}) z^{-1} + (a_z + b_2 \frac{\hat{\alpha}}{\hat{\beta}}) z^{-2}} \quad . \tag{2.19}$$

In the appendix, it is shown that a test reference output $y_m^*(k)$ must have less than $(m+d+n)/2$ frequencies. For this system it is sufficient to let $y_m^*(k)$ contain a single frequency. Hence, $y_m^*(k)$ can be represented by

$$y_m^*(k) = e^{j\Omega k} = \cos(\Omega k) + j \sin(\Omega k) \quad . \tag{2.20}$$

Given Ω, the controller parameters which satisfy the tuning condition can be found by substituting Equation (2.21) into (2.19) and setting $T_y(z^{-1})$ equal to unity. After some manipulations, the parameters are found to be

$$\alpha^*(\Omega) = \frac{c_1 c_6 - c_3 c_4}{c_5 c_1 - c_2 c_4} \tag{2.21}$$

$$\beta^*(\Omega) = \frac{c_5 c_3 - c_6 c_2}{c_5 c_1 - c_2 c_4} \tag{2.22}$$

with

$$c_1 = a_1 \sin(\Omega) + a_2 \sin(2\Omega)$$

$$c_2 = b_1 \sin(\Omega) + b_2 \sin(2\Omega)$$

$$c_3 = b_2 \sin(\Omega)$$

$$c_4 = 1 + a_1 \cos(\Omega) + a_2 \cos(2\Omega)$$

$$c_5 = b_1 \cos(\Omega) + b_2 \cos(2\Omega)$$

$$c_6 = b_1 + b_2 \cos(\Omega) \quad .$$

For $y_m^*(k)$ to be a test reference output, the poles of $T_y(z^{-1})$ must be stable. The characteristic equation for $T_y(z^{-1})$ is

$$P(z^{-1}) = 1 + \left(a_1 + b_1 \frac{\alpha^*(\Omega)}{\beta^*(\Omega)}\right) z^{-1} + \left(a_2 + b_2 \frac{\alpha^*(\Omega)}{\beta^*(\Omega)}\right) z^{-2} \quad . \tag{2.23}$$

It is important to note that because $\alpha^*(\Omega)$ and $\beta^*(\Omega)$ appear only as a ratio in Equation (2.23), the stability of $T_y(z^{-1})$ is determined by this ratio. The stability of $P(z^{-1})$ can be determined by using the Jury test.

Example 2.1: In this example, the nominal system is simulated. The following plant coefficients are used:

$$a_1 = -0.2$$

$$a_2 = -0.99$$

$$b_1 = 1.0$$

$$b_2 = 1.5 \quad .$$

These coefficients produce plant poles at 1.1 and -0.9 and a plant zero at 1.5. With $\Omega = 0.4, \hat{\alpha} = \hat{\beta}$ were calculated to be 1.3503 and 1.12475, respectively. With these parameters, $T_y(z^{-1})$ has poles at $-0.3165 \pm j\,0.3991$. Clearly, $T_y(z^{-1})$ is stable and y_m^* is a test reference output. Notice that even though the plant is nonminimin phase, a test reference output exists. This shows Assumption A1 does not imply the plant must be nonminimum phase.

This system was simulated with $\hat{\alpha}(0) = \hat{\beta}(0) = 1.0$, $\varepsilon = 1.0$, $\Omega = 0.4$ and the plant initial conditions at zero. The trajectories of $\hat{\alpha}(k)$ and $\hat{\beta}(k)$ are shown in Figure 2.1. In this disturbance-free case the parameters converged to their correct values nicely.

Unless stated otherwise, the system used in Example 2.1 will be used in the following examples with the same parameters and initial conditions.

Example 2.2: In this example, a disturbance is added through the reference model input. This disturbance was denoted by v(k) in the analysis and will be a single sinusoid in this example. For $\Omega = 2.5$, Equations (2.21) and (2.22) yields an $\hat{\alpha}(\Omega)$ and $\hat{\beta}(\Omega)$ of -2.24091 and -1.1866, respectively. Substituting these parameters into Equation (2.19) yields an unstable $T_y(z^{-1})$, so this frequency cannot be a test reference output. In the second simulation, v(k) was set to be 0.25 cos(2.5k). The trajectories of $\hat{\alpha}(k)$ and $\hat{\beta}(k)$ are shown in Figure 2.2. As expected, the parameter values do not converge, since the controller does not have enough freedom to satisfy matchability.

Example 2.3: This example is the first of three examples to demonstrate the versatility of Assumption A2 in modelling parasitics, nonlinearities, etc.

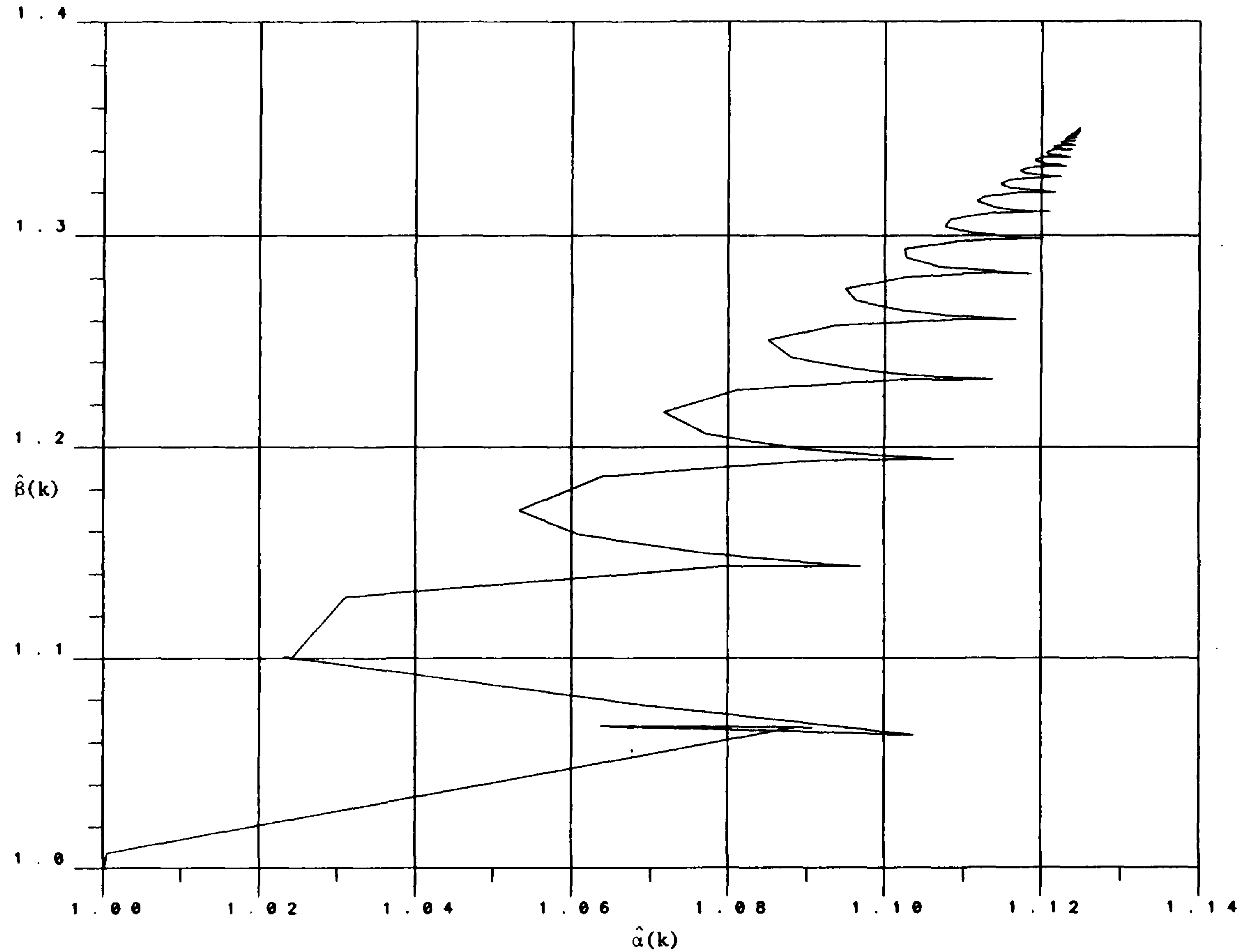

Figure 2.1. Example 2.1, $0 \leq k \leq 500$.

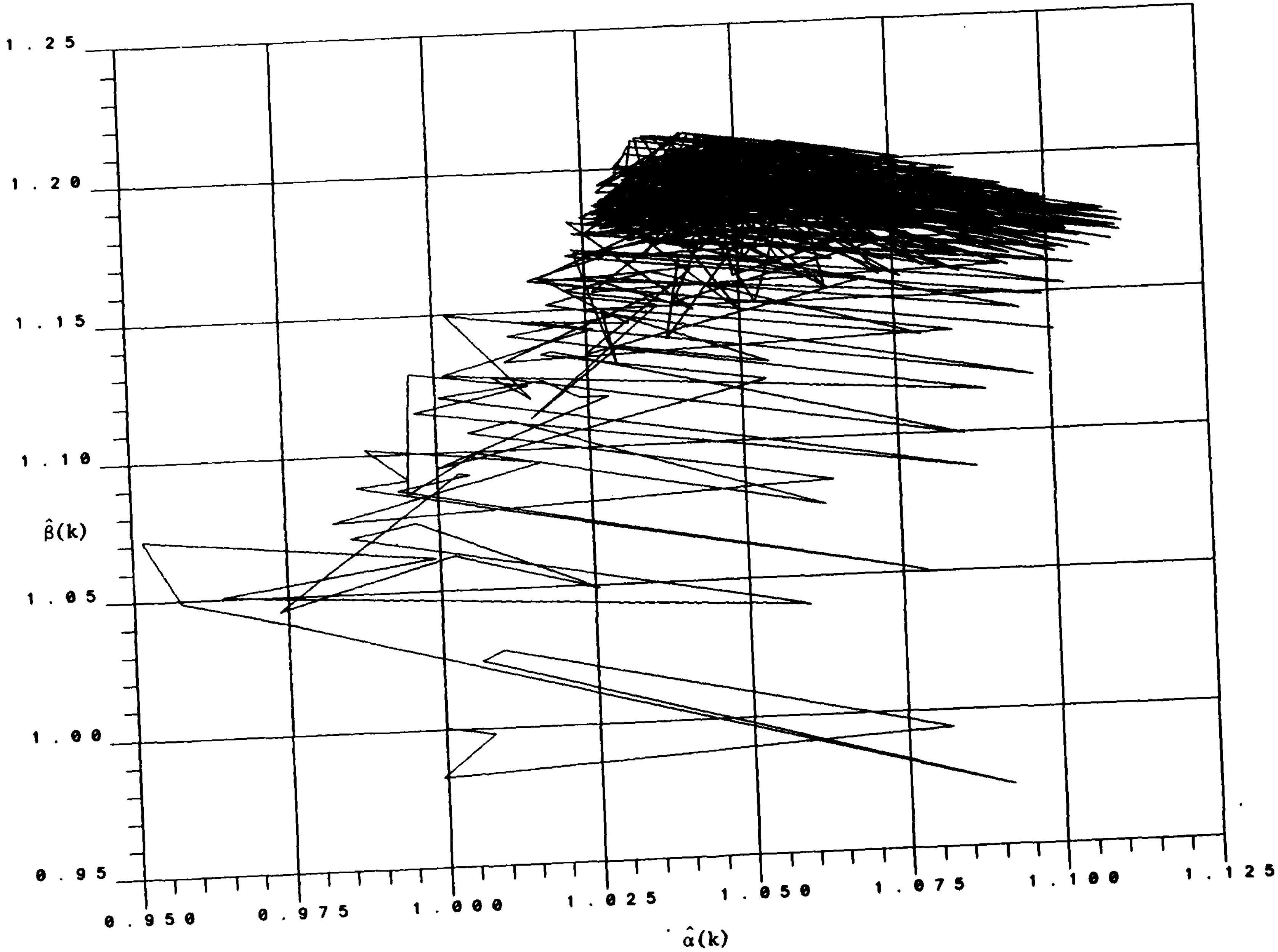

Figure 2.2. Example 2.2, $0 \leq k \leq 500$.

In this example the disturbance considered is an unmodelled pole near the origin. Let the nominal plant be represented by

$$A(q^{-1})y(k) = B(q^{-1})u(k) \quad ,$$

and the perturbed plant by

$$A(q^{-1})(1 + \delta q^{-1})y(k) = B(q^{-1})u(k) \tag{2.24}$$

where δ is some small constant.

Equation (2.24) can be manipulated into

$$A(q^{-1})y(k) = B(q^{-1})u(k) - \delta q^{-1}A(q^{-1})y(k) \quad . \tag{2.25}$$

The perturbed plant can be written as

$$A(q^{-1})y(k) = B(q^{-1})u(k) + w(k) \quad , \tag{2.26}$$

where $w(k)$ is

$$w(k) = - \delta q^{-1}A(q^{-1})y(k) \tag{2.27}$$

$$w(k) = - \delta \sum_{i=1}^{n_A} a_i \; y(k-i) \quad . \tag{2.28}$$

Introducing $0 < \mu < 1$, and using Cauchy-Schwarz,

$$|w(k)| \leq \delta \max|\mu^{i-k} a_i| \sum_{i=1}^{n_A} |y(k-i)|\mu^{k-i} \quad .$$

This expression shows clearly that a disturbance introduced by an unmodelled pole near the origin satisfies Assumption A2. For the simulation, a pole at 0.2 was placed into the plant. The trajectories of the parameters for this example are shown in Figure 2.3.

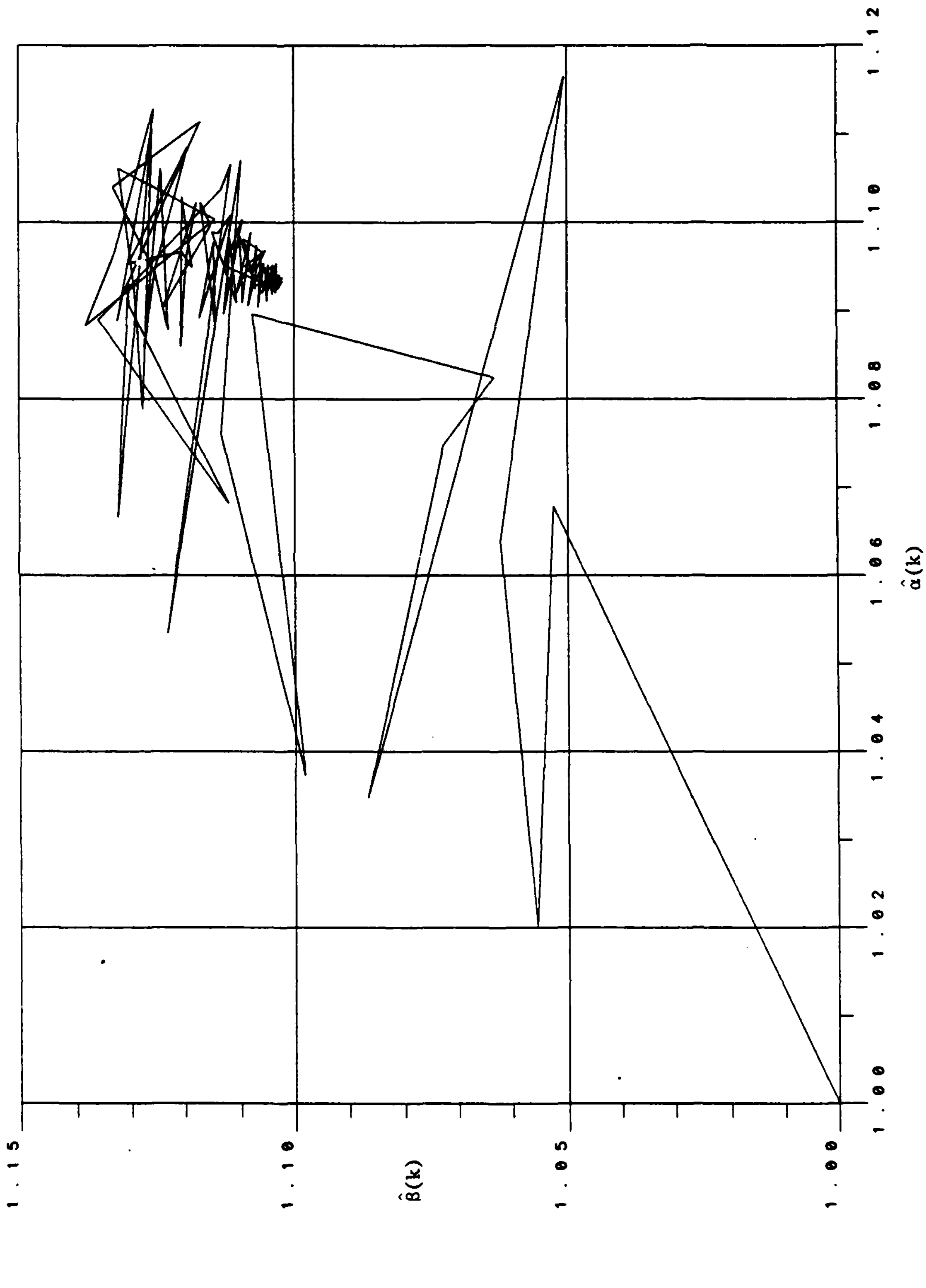

Figure 2.3. Example 2.3, $0 \le k \le 500$.

Example 2.4: In this example, some nonlinearities are shown to satisfy Assumption A2. To simplify the analysis, assume the plant with a small nonlinearity can be written as

$$A(q^{-1})y(k) = q^{-1}B(q^{-1})f(u(k)) \tag{2.29}$$

$$A(q^{-1})y(k) = q^{-1}B(q^{-1})u(k) + q^{-1}B(q^{-1})(f(u(k)) - u(k)) \quad . \tag{2.30}$$

This implies $w(k)$ is

$$w(k) = q^{-1}B(q^{-1})(f(u(k)) - u(k) \quad . \tag{2.31}$$

Using the Cauchy-Schwarz inequality,

$$|w(k)| \le \sum_{i=1}^{n_B} |f(u(k)) - u(k)||b_i| \quad . \tag{2.32}$$

Introducing $0 < \mu < 1$,

$$|w(k)| \le \sum_{i=1}^{n_B} \mu^{k-i}|f(u(k)) - u(k)||\mu^{i-k}\ b_i| \tag{2.33}$$

$$|w(k)| \le \max|\mu^{i-k}\ b_i| \sum_{i=1}^{n_B} \mu^{k-i}|f(u(k)) - u(k) \quad .$$

If

$$|f(u(k)) - u(k)| < \delta|u(k)| + w \quad ,$$

then $w(k)$ will satisfy Assumption A2. For the simulation, $f(u(k))$ was defined as

$$f(u(k)) = \frac{u(k) + 0.2u(k)^3}{1 + u(k)^2} \quad .$$

The parameter trajectories are given in Figure 2.4.

Example 2.5: To conclude this section, a small time variation in the parameters is shown to be encompassed by Assumption 2. For this example let the plant be represented by

$$y(k) = q^{-1}B(q^{-1})u(k) - \sum_{i=1}^{n_A} a_i y(k-i)(1 + 0.2 \sin(\Omega_s k) \quad , \qquad (2.34)$$

where Ω_s is the frequency of the parameters. Clearly, a slight rearrangement of Equation (2.34) yields

$$w(k) = - 0.2 \sum_{i=1}^{n_A} a_i y(k-i) \sin(\Omega_s k) \quad .$$

This equation is very similar to Equation (2.28). Hence, w(k) satisfies Assumption A2. Again, the parameter trajectories are presented in Figure 2.5.

2.5. Discussion

The analysis presented has shown the local stability of the adaptive system about a tuned solution. This stability was found to be robust to a broad class of disturbances in both the plant and the reference output. Through the use of several examples, it was demonstrated that bounded disturbances, neglected poles or zeros near the origin, slight nonlinearities and some time variations satisfy Assumption A2.

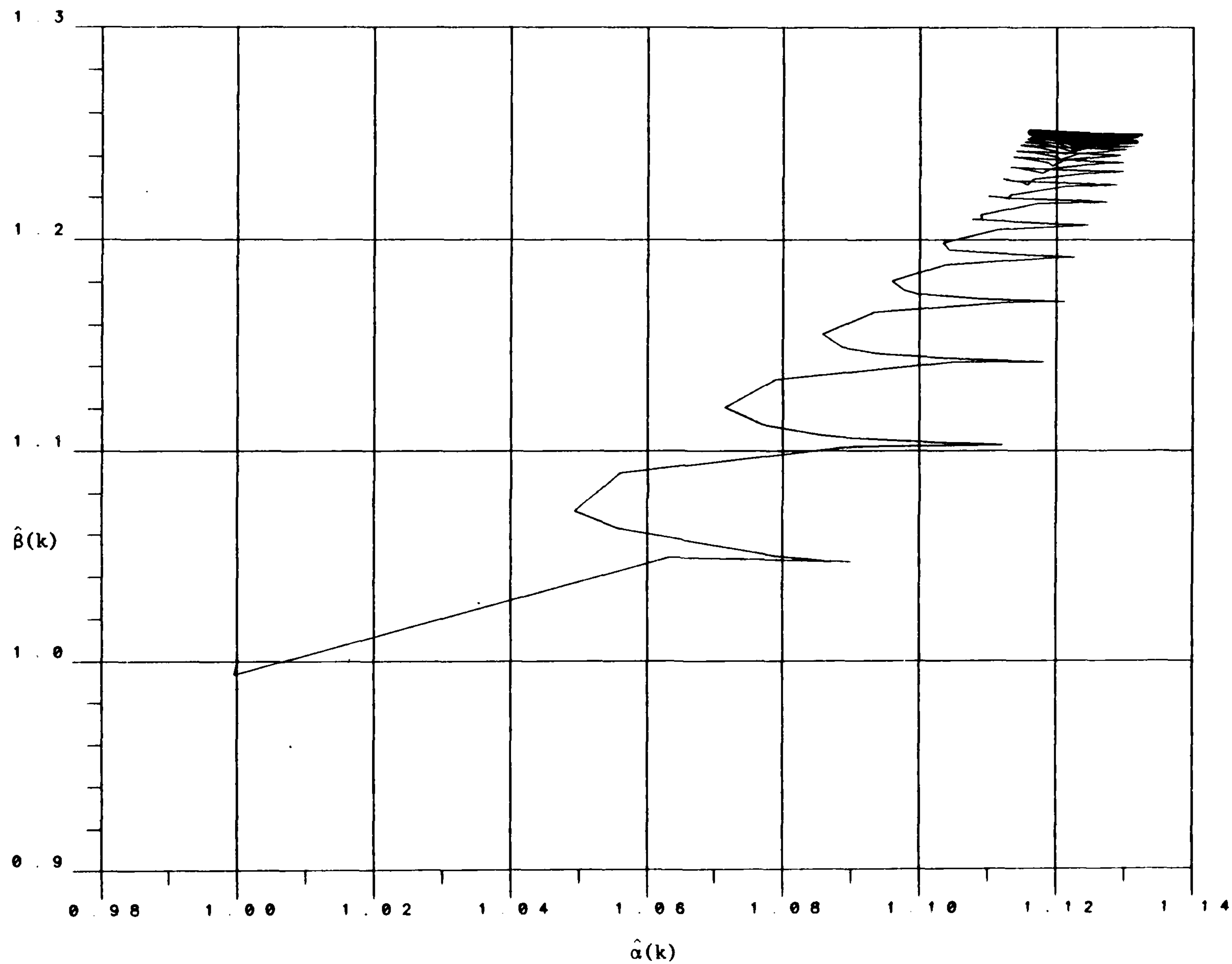

Figure 2.4. Example 2.4, $0 \leq k \leq 500$.

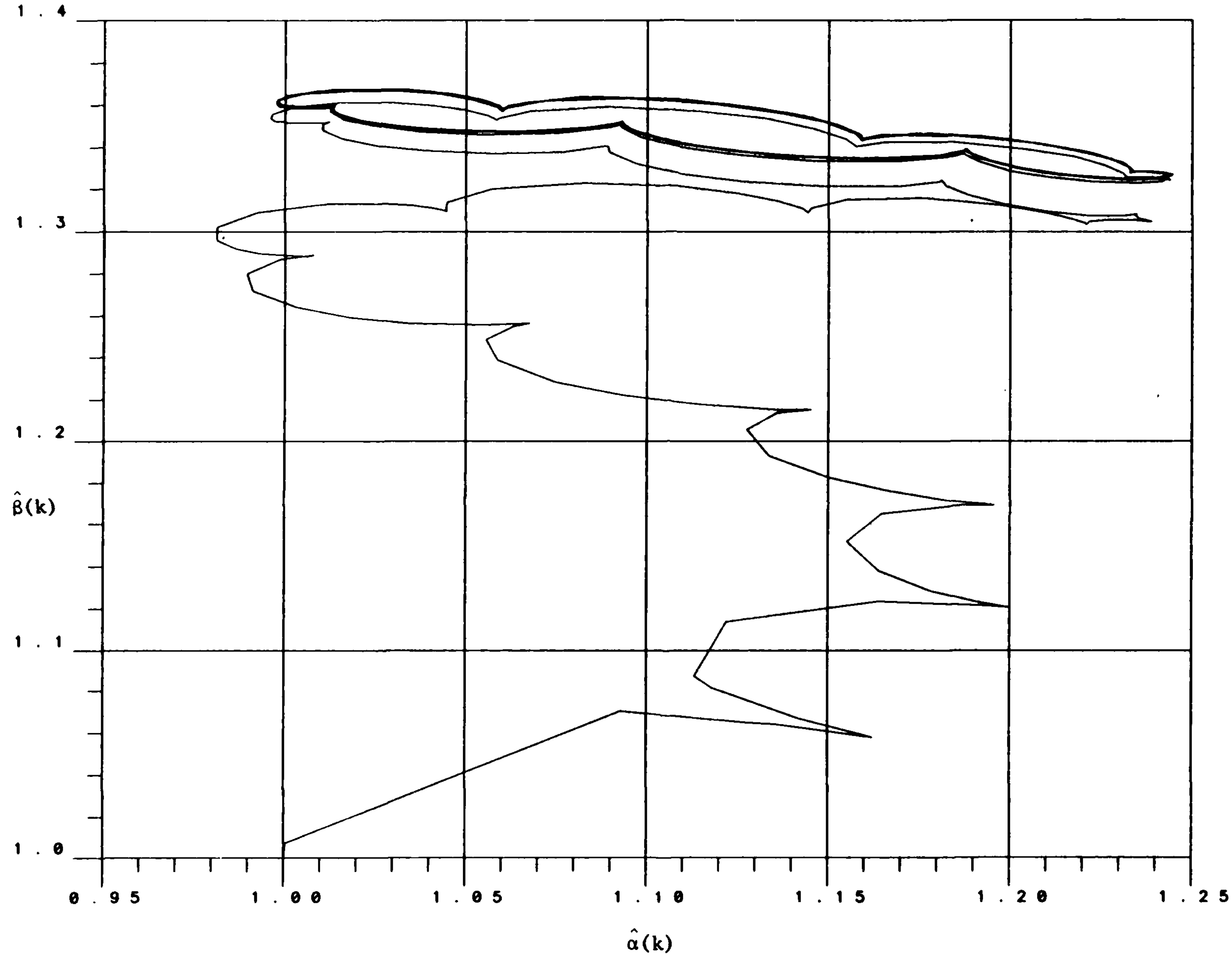

Figure 2.5. Example 2.5, $0 \le k \le 500$.

Also, during the simulations, several characteristics of the system were noticed. First, the system exhibited very strong stability properties. With ε large and with large disturbances the simulations remained stable. These properties may be due to the fact that the plant used in the simulation was only second order. One characteristic second order systems have that higher order systems do not have is that the set of characteristic equation coefficients which yield a stable system is convex. Another characteristic of the simulations was that some disturbances seemed to shift the equilibrium parameter values. Perhaps a better understanding of this phenomena would give some more insight into the system performance.

CHAPTER 3

A DIRECT ADAPTIVE SCHEME FOR SAMPLED DATA SYSTEMS

In this chapter a direct adaptive scheme suitable for a sampled continuous process is presented. As mentioned in the introduction, as the sampling interval is reduced to zero, the zeros of the equivalent discrete model approach known values. The adaptive algorithm proposed assumes the unstable zeros are exactly known, and to simplify the analysis, the plant is assumed linear and disturbance free. With these assumptions, a parametric model is derived to retain the unstable zeros in the tracking transfer function. These parameters are estimated using a least squares algorithm augmented by projecting the parameters onto a convex set. Global convergence for the algorithm is established via an analysis similar to that used in Goodwin et al. (1980). The chapter concludes with a discussion of the weaknesses and strengths of the algorithm.

3.1. System Reparametrization

To establish the notation, the aforementioned assumptions are precisely stated.

Assumption A4: The sampled-data system may be represented by

$$A(q^{-1})y(t) = q^{-1}B_u(q^{-1})B(q^{-1})u(t) \tag{3.1}$$

where $A(q^{-1})$, $B(q^{-1})$ and $B_u(q^{-1})$ are polynomials in the backward shift operator q^{-1}; $A(q^{-1})$ is monic and $A(q^{-1})$ and $B_u(q^{-1})$ are relatively prime;

$B(q^{-1})$ describes the stable zeros and the sign of its leading coefficient is known (say, positive); $\{y(t)\}$ and $\{u(t)\}$ are the output and input sequences, respectively, of the sampled system.

Assumption A5: Integers n and m are known such that

$$n \geq \deg A(q^{-1}) \quad \text{and} \quad m \geq \deg B(q^{-1}) \qquad .$$

Assumption A6: $B_u(q^{-1})$ is known, has degree d, and may include the system delays. The unstable zeros represented by $B_u(q^{-1})$ will be denoted by z_{ui}, where $i = 1$ to d. For the purposes of this paper, we shall assume that all z_{ui} are real.

With these assumptions the parametric model given by Johansson in Chapter 5 of Johansson (1983) is derived.

In order to maintain consistency with the continuous time approach, observer poles are introduced. Let

$$N = \max\{n, m + d + 1\}$$

be the number of such poles. Let $C(q^{-1})$ and $T(q^{-1})$, of degree $d+1$ and N, respectively, be polynomials with stable zeros. The relative primeness of $A(q^{-1})$ and $B_u(q^{-1})$ implies that there exist polynomials $\bar{S}(q^{-1})$ and $R(q^{-1})$ of degree N-m and N-1, respectively, such that

$$A(q^{-1})\bar{S}(q^{-1}) + q^{-1}B_u(q^{-1})q^{-1}R(q^{-1}) = C(q^{-1})T(q^{-1}) \qquad . \tag{3.2}$$

Applying (3.2) to y(t) yields

$$C(q^{-1})T(q^{-1})y(t) = q^{-1}(S(q^{-1})B_u(q^{-1})u(t) + q^{-1}R(q^{-1})B_u(q^{-1})y(t)) \quad , \tag{3.3}$$

where $S(q^{-1}) = \bar{S}(q^{-1})B(q^{-1})$ is of degree less than or equal to N and has leading coefficient b_o strictly positive. Define a regression vector $\phi(t)$ by

$$T(q^{-1})\phi(t) = [u(t) \ldots u(t-N)y(t-1) \ldots y(t-N)] \quad . \tag{3.4}$$

Define a parameter vector θ by (with $s_o = b_o$):

$$\theta = [s_o \ldots s_N \; r_1 \ldots r_N] \tag{3.5}$$

where $s_i (i = 0 \text{ to } N)$ are the coefficients of $S(q^{-1})$ and $r_j (j = 1 \text{ to } N)$ are the coefficients of $R(q^{-1})$.

Then, if the roots of $T(q^{-1})$ are stable and all initial conditions are zero,

$$C(q^{-1})y(t) = \theta^T B_u(q^{-1})\phi(t-1) \quad . \tag{3.6}$$

3.2. A Direct Adaptive Algorithm

Let $C(q^{-1})$ and $T(q^{-1})$ have stable roots, and let the objective be to match

$$C(q^{-1})y(t) = q^{-1}B_u(q^{-1})r(t) \quad . \tag{3.7}$$

Rearrangement of (3.6) yields

$$C(q^{-1})y(t) = B_u(q^{-1})\theta^T\phi(t-1) \quad , \tag{3.8}$$

which implies that the control law is

$$r(t) = \theta^T \phi(t) \quad . \tag{3.9}$$

Examination of (3.2) will show that, since $C(q^{-1})$ and $T(q^{-1})$ are stable, $\bar{S}(q^{-1})$ and $B_u(q^{-1})$ are relatively prime. This implies that $S(q^{-1})$ and $B_u(q^{-1})$ are prime. This fact motivates the next assumption.

<u>Assumption A7</u>: A convex set $\mathbf{C} \subset \mathbb{R}^N$ is known such that

1. the vector of coefficients of $S(q^{-1})$ belongs to $\mathbf{C}$, and
2. for any polynomial $S'(q^{-1})$ corresponding to any vector in $\mathbf{C}$,

$$|S'(z_{ui}^{-1})| > \varepsilon \quad , \quad 1 \le i \le d$$

and

$$\lim_{z \to \infty} |S'(z^{-1})| = |b_o'| > \varepsilon \quad .$$

Examination of the regression vector $\phi(t)$, parameter vector θ, and control law (3.9) reveals the fact that $S(q^{-1})$ will be the denominator of the controller. Since this controller is a sampled data version of a continuous-time controller, as the sampling period goes to zero, the poles of this controller tend to 1; moreover, the zeros of the system, z_{ui}, are in $(-\infty,-1]$. The convex set $\mathbf{C}$ of assumption, therefore, can be described by the following inequalities (using $b_o > \varepsilon$):

$$S'(z_{ui}^{-1}) \ge \varepsilon \quad , \quad 1 \le i \le d$$

$$\lim_{z \to \infty} S'(z^{-1}) = b_o' \ge \varepsilon \quad .$$

Now, since $R(q^{-1})$ and $S(q^{-1})$ are unknown, an adaptive control algorithm is proposed.

Let $\hat{\theta}(t)$ be an estimate of θ and define $\varphi(t)$ as

$$\varphi(t) = B_u(q^{-1})\phi(t) \qquad . \tag{3.10}$$

The algorithm is

$$\hat{\theta}(t) = \mathcal{P}_{\mathcal{C}}\left(\hat{\theta}(t-1) + \frac{P(t-2)\varphi(t-1)}{1 + \varphi(t-1)^T P(t-2)\varphi(t-1)} \left(C(q^{-1})y(t) - \hat{\theta}(t-1)^T \varphi(t-1)\right)\right) \tag{3.11}$$

where $\mathcal{P}_{\mathcal{C}}$ is the projection on $\mathcal{C}$ proposed on page 92 of Goodwin and Sin (1984);

$$P(t-1) = P(t-2) - \frac{P(t-2)\varphi(t-1)\varphi(t-1)^T P(t-2)}{1 + \varphi(t-1)^T P(t-2)\varphi(t-1)} \quad , \tag{3.12}$$

$$P(0) > 0 \quad ;$$

$$r(t) = \hat{\theta}(t)^T \phi(t) \qquad . \tag{3.13}$$

The use of the projection $\mathcal{P}_{\mathcal{C}}$ guarantees that the polynomial $\hat{S}(q^{-1},t)$ obtained from $\hat{\theta}(t)$ satisfies the condition that

$$|\hat{S}(z_{ui}^{-1},t)| > \varepsilon \quad , \quad 1 \le i \le d$$

and

$$\lim_{z \to \infty} |\hat{S}(z^{-1},t)| > \varepsilon \quad .$$

3.3. Convergence of the Algorithm

The following theorem gives the properties of the algorithm.

Theorem 3: Under the Assumptions A4 through A7, the algorithm described by (3.11), (3.12), and (3.13) has the properties

1. $\hat{\theta}(t)$ is bounded,

2. $u(t)$ and $y(t)$ are bounded for bounded $r(t)$, and

3. $\lim_{t\to\infty}[C(q^{-1})y(t) - q^{-1}B_u(q^{-1})r(t)] = 0.$

Proof: We shall use the techniques of proof introduced by Goodwin, Ramadge and Caines (1980). The presence of $B_u(q^{-1})$, however, will impose some additional difficulties. We shall first state some technical lemma stating the properties of the estimation used in the algorithm.

Lemma 1: Under the aforementioned assumptions,

1. $\|\hat{\theta}(t` - \theta\| \le K_1,$ for all t;
2. $\lim_{t\to\infty}|C(q^{-1})y(t) - \hat{\theta}(t)^T\varphi(t-1)| = 0;$
3. $\lim\|\hat{\theta}(t) - \hat{\theta}(t-1)\| = 0.$

Proof: See Goodwin and Sin (1984) Lemma 3.3.6.

Lemma 2: Let $P(q^{-1},t)$ and $Q(q^{-1},t)$ be polynomials with degree n_p, n_q, $q_j(t)$ ($j = 0$ to n_q) such that $p_i(t)$ and $q_j(t)$ are bounded and satisfy

$$\lim_{t\to\infty}|q_j(t) - q_j(t-1)| = 0 \qquad .$$

For any sequence $v(t)$, let $F(P,Q,v,t)$ be the sequence defined by

$$F(P,Q,v,t) = P(q^{-1},t)[Q(q^{-1},t)u(t)] - [P(q^{-1},t)Q(q^{-1},t)]u(t) \quad , \tag{3.14}$$

where the first term on the right-hand side implies $P(q^{-1},t)$ operates on $Q(q^{-1},t)u(t)$, while the second term implies that the product $[P(q^{-1},t)$ $A(q^{-1},t)]$ operates on $u(t)$. Then for any $\varepsilon > 0$, there exists a T such that, for all $t > T$

$$|F(P,Q,v,t)| \le \varepsilon\underline{P}n_p(n_p - 1)n_Q \sup_{0\le \ell \le n_p+n_Q} |v(t-\ell)| \tag{3.15}$$

where

$$\underline{P} > |p_i(t)| \qquad \forall t, 0 \leq i \leq n_p \qquad .$$

Proof: See Goodwin and Sin (1984) Lemma 4.3.2.

Lemma 3: If

$$|\hat{S}(z_{ui}^{-1},t)| > \varepsilon \quad , \qquad 1 \leq i \leq d \quad ,$$

and

$$\lim_{z\to\infty} |\hat{S}(z^{-1},t)| > \varepsilon \qquad ,$$

then there exist time-varying polynomials $\alpha(q^{-1},t)$ and $\beta(q^{-1},t)$ such that

$$[\alpha(q^{-1},t)\hat{S}(q^{-1},t)] + [\beta(q^{-1},t)q^{-1}B_u(q^{-1})] = 1$$

and the coefficients of $\alpha(q^{-1},t)$ and $\beta(q^{-1},t)$ are bounded.

Proof: The hypothesis on $\hat{S}(q^{-1},t)$ implies that $\hat{S}(q^{-1},t)$ and $q^{-1}B_u(q^{-1})$ are relatively prime for any t. Existence of $\alpha(q^{-1},t)$ and $\beta(q^{-1},t)$ follows from the Bezout identity; moreover, the Bezoutian of $\hat{S}(q^{-1},t)$ and $q^{-1}B_u(q^{-1})$ is (see Kailath (1980), Example 2.4-17, page 159)

$$\hat{s}_o(t) \cdot \prod_{i=1}^{d} \hat{S}(z_{ui}^{-1},t) \qquad ,$$

which is uniformly bounded from below. This implies the boundedness of the coefficients of $\alpha(q^{-1},t)$ and $\beta(q^{-1},t)$.

Before proceeding further, l s note that there is no finite escape time, since the mapping from t to t+1 has no singularity. In particular, the projection operation in the control algorithm implies that no division by zero will occur. The proof of the theorem will now be done using the small gain theorem:

<u>Forward path: $u(t) \to y(t)$</u>: Let $\eta(t)$ be defined by

$$\eta(t) = C(q^{-1})y(t) - \hat{\theta}(t)^T\varphi(t-1)$$

$$= C(q^{-1})y(t) - \hat{\theta}(t)^T B_u(q^{-1})\phi(t-1) \quad . \tag{3.16}$$

Adding and subtracting $B_u(q^{-1})r(t-1)$ to (3.16) and using the definition of the control law (3.13) yields

$$C(q^{-1})y(t) = \eta(t) + q^{-1}B_u(q^{-1})r(t) + \hat{\theta}(t)^T[B_u(q^{-1})\phi(t-1)]$$

$$- B_u(q^{-1})[\hat{\theta}(t-1)^T\phi(t-1)] \quad . \tag{3.17}$$

Now, for any ε, let T_o be a finite time given by Lemmas 1 and 2, such that

$$|\eta(t)| < \varepsilon \qquad \text{for all } t \geq T_o \tag{3.18}$$

and

$$\sup_{T \geq t \geq T_o} |C(q^{-1})y(t)| \leq \varepsilon + K_2 \sup_{T \geq t \geq T_o} |B_u(q^{-1})r(t-1)| + \varepsilon K_3 \sup_{T \geq t \geq T_o} \|\phi(t-1)\| , \tag{3.19}$$

for some constants K_2 and K_3. The finite nature of the ℓ_∞ gains of $C(q^{-1})^{-1}$ and $T(q^{-1})^{-1}$ then imply that

$$\sup_{T \geq t \geq T_o} |y(t)| \leq \varepsilon + \varepsilon K_4 \sup_{T \geq t \geq T_o} |u(t-1)| + \varepsilon K_5 \sup_{T \geq t \geq T_o} |y(t-1)| + K_6 \quad , \tag{3.20}$$

for some constants K_4, K_5 and K_6. The fact that there is no finite escape time then implies that

$$\sup_{T \geq t \geq T_o} |y(t)| \leq \frac{\varepsilon K_4}{1 - \varepsilon K_5} \sup_{T \geq t \geq T_o} |u(t)| + K_7 \tag{3.21}$$

for some K_7.

<u>Feedback path: $y(t) \rightarrow u(t)$</u>:

Let $\alpha(q^{-1},t)$ and $\beta(q^{-1},t)$ be two time varying polynomials defined below. Applying $\beta(q^{-1},t)$ to (1) and applying $\alpha(q^{-1},t)B(q^{-1})T(q^{-1})$ to (3.13) result in the following two equations:

$$[\beta(q^{-1},t)A(q^{-1})]y(t) = [\beta(q^{-1},t)B(q^{-1})q^{-1}B_u(q^{-1})]u(t) \tag{3.22}$$

$$[\alpha(q^{-1},t)B(q^{-1})T(q^{-1})]r(t) = [\alpha(q^{-1},t)B(q^{-1})T(q^{-1})]\hat{\theta}(t)^T\phi(t) \quad . \tag{3.23}$$

Let $\hat{R}(q^{-1},t)$ and $\hat{S}(q^{-1},t)$ be defined as those polynomials whose respective coefficients are those coefficients in $\hat{\theta}(t)$. With $\alpha(q^{-1},t)$, $\beta(q^{-1},t)$ related to $\hat{R}(q^{-1},t)$ and $\hat{S}(q^{-1},t)$ by Lemma 3, Equation (3.23) may be rewritten as

$$\begin{aligned}[\alpha(q^{-1},t)B(q^{-1})T(q^{-1})]r(t) &= [\alpha(q^{-1},t)B(q^{-1})][\hat{S}(q^{-1},t)u(t) + \hat{R}(q^{-1},t)y(t) \\ &\quad + T(q^{-1})[\hat{\theta}(t)^T\phi(t)] - \hat{\theta}(t)^T[T(q^{-1})\phi(t)]] \quad .\end{aligned} \tag{3.24}$$

Summing respective sides of (3.22) and (3.24) and applying Lemma e yields the equality

$$\begin{aligned}&[\alpha(q^{-1},t)B(q^{-1})T(q^{-1})]r(t) + [\beta(q^{-1},t)A(q^{-1})]y(t) = B(q^{-1})u(t) \\ &\quad + [\alpha(q^{-1},t)B(q^{-1})][\hat{R}(q^{-1},t)y(t) + T(q^{-1})[\theta(t)^T\phi(t) \\ &\quad - \hat{\theta}(t)^T[T(q^{-1})\phi(t)]] + [\alpha(q^{-1},t)B(q^{-1})][\hat{S}(q^{-1},t)u(t)] \\ &\quad - [\alpha(q^{-1},t)B(q^{-1})\hat{S}(q^{-1},t)]u(t)\end{aligned} \tag{3.25}$$

or

$$
\begin{aligned}
B(q^{-1})u(t) = {} & [\alpha(q^{-1},t)B(q^{-1})T(q^{-1})]r(t) + [\beta(q^{-1},t)A(q^{-1})]y(t) \\
& - [\alpha(q^{-1},t)B(q^{-1})][\hat{R}(q^{-1},t)y(t) + T(q^{-1})[\hat{\theta}(t)^T\phi(t)] \\
& - \hat{\theta}(t)^T[T(q^{-1})\phi(t)]] + [\alpha(q^{-1},t)B(q^{-1})][\hat{S}(q^{-1},t)u(t)] \\
& - [\alpha(q^{-1},t)B(q^{-1})\hat{S}(q^{-1},t)]u(t) \qquad . \qquad (3.26)
\end{aligned}
$$

$B(q^{-1})^{-1}$ has a finite ℓ_∞-gain and we have no escape time; with the lemmas we can find a T_1 such that

$$
\begin{aligned}
\sup_{T\geq t\geq T_1} |u(t)| \leq K_8 + K_9 \sup_{T\geq t\geq T_1} |y(t)| + \varepsilon K_{10} \sup_{T\geq t\geq T_1} \| T(q^{-1})\phi(t)\| \\
+ \varepsilon K_{11} \sup_{T\geq t\geq T_1} |u(t)| \qquad (3.27)
\end{aligned}
$$

for some constants K_8, K_9, K_{10} and K_{11}.

Since $T(q^{-1})^{-1}$ has a finite ℓ_∞-gain, Equation (3.27) may be rewritten as

$$
\sup_{T\geq t\geq T_1} |u(t)| \leq K_{12} + K_{13} \sup_{T\geq t\geq T_1} |y(t)| + \varepsilon K_{14} \sup_{T\geq t\geq T_1} |u(t)| \qquad (3.28)
$$

for some constants K_{12}, K_{13} and K_{14}. Equation (3.28) implies that

$$
\sup_{T\geq t\geq T_1} |u(t)| \leq \frac{K_{13}}{1 - \varepsilon K_{14}} \sup_{T\geq t\geq T_1} |y(t)| + K_{15} \qquad (3.29)
$$

for some constant K_{15}.

Closing the loop: The boundedness of u(t) and y(t) follows from application of the Small Gain Theorem to (3.21) and (3.29) with a choice of ε such that

$$\frac{\varepsilon K_4}{1 - \varepsilon K_5} \cdot \frac{K_{13}}{1 - \varepsilon K_{14}} < 1 \quad .$$

We may now conclude from (3.17) that

$$\lim_{t\to\infty} [c(q^{-1})y(t) - q^{-1}B_u(q^{-1})r(t)] = 0 \quad .$$

3.4. Discussion

As mentioned previously, the stability analysis in Section 3.3 is similar to the proof presented in Goodwin et al. (1980). As a result, Theorem 2 has many of the same restrictive assumptions which handicap the Goodwin, Ramadge and Caines stability result. In particular, Assumptions A4 and A5 of Theorem 2 limit the analysis to disturbance-free linear time invariant plants with known order. Since most industrial processes do not meet these assumptions, further work is needed to relax these assumptions.

Assumptions A6 and A7 are unique to this algorithm and are required to cope with the nonminimum phase zeros. Assumption A6 assumes knowledge of the unstable zero locations, while Assumption A7 enables the projection operator in the algorithm to prevent the controller from cancelling the unstable zeros. Although the nonminimum phase zeros are known as the sampling rate tends to zero, practical considerations will limit the speed of sampling and, the zero locations will be unknown. This limitation motivates future research into the robustness of the algorithm with respect to uncertainties in the zero locations.

Although the analysis applies only to a nominal plant, Theorem 2 represents a very strong stability result. This result states that not only will the parameters and signals remain bounded, but also, the tracking error will tend to zero for all initial conditions. Another important feature of the analysis is that the proof of Theorem 2 relies upon the convergence properties of the estimator instead of its structure. Therefore, if the least squares estimator is replaced by any scheme for which Lemma 1 is valid, Theorem 2 will hold. This is very important in applications, because often the least squares estimator is modified to compensate for slowly varying plants or disturbances. Goodwin and Sin (1980) list many common modifications and their convergence properties.

CHAPTER 4

CONCLUSIONS

In this thesis two stability results are developed for one step ahead adaptive controllers. In Chapter 2 an analysis is presented of the local behavior of a one step ahead adaptive controller around a tuned solution. Instead of placing assumptions on the order, delay or minimum phase properties of the plant, the existence of a unique tuned solution is assumed. This implies that for a particular reference model output there exists a unique stabilizing parameter vector such that the plant output will track the reference model output exactly. Assuming this tuned solution exists, the closed-loop system was shown to be locally stable, and small departures from the nominal do not disturb this stability.

In the appendix the existence of a unique tuned solution is shown to be equivalent to the classical persistence of excitation. Furthermore, the examples of Section 2.4 have shown that a unique tuned solution does not imply the plant is minimum phase. It is clear that the assumption of the existence of a unique tuned solution has not been fully analyzed, and further research is needed to ascertain all the implications of such an assumption.

The second stability result presented in this thesis deals with the problems associated with direct adaptive control of a sampled data process. Even if a continuous time plant is minimum phase, sampling will often introduce nonminimum phase zeros in the equivalent discrete model. These nonminimum phase zeros cannot be cancelled if system stability is

to be maintained. Johansson (1983) has shown that the minimum a priori knowledge needed is the unstable zero locations. Fortunately, Astrom et al. (1984) have estabilished the location of these zeros for very rapidly sampled systems. Using this information, the algorithm presented in Section 3.2 retains the unstable zeros in the tracking transfer function.

The analysis presented in Section 3.3 represents the first step towards establishing a robust direct adaptive scheme for sampled data systems. Assuming the zero locations are known, the analysis in Section 3.3 shows that the system will have bounded signals and parameters and that the tracking error will go to zero. Unfortunately, in practical situations the sampling rate is limited, and therefore, the exact zero locations are not known. Furthermore, industrial processes are seldom disturbance-free.

These weaknesses in the analysis motivate further research into the robustness of the algorithm with respect to plant disturbances and errors in the unstable zero locations.

APPENDIX

PROPERTIES OF THE TUNED SOLUTION

The test reference output $y_m^*(k)$ was defined to be N-periodic, as a consequence it can be represented as

$$y_m^*(k) = \sum_{i=-N_1}^{N_2} y_{m_i}^* \, z_i^k \quad , \qquad \forall k$$

with

$$N_1 \leq \frac{1}{2} N \ , \ N_2 \leq \frac{1}{2} (N - 1) \ , \ z_i = \exp(j\frac{2\Pi i}{N}) \ , \quad y_{m,-i}^* = \bar{y}_{mi}^*$$

where $\bar{y}_{mi}^*$ is the complex conjugate of y_{mi}^*.

In the following we denote by E the set

$$E = \{i : y_{mi}^* \neq 0\} \qquad .$$

The number of elements in E is denoted by n_E Θ^* can be decomposed into

$$\Theta^* = (\beta^*, \beta^*, \ldots, \beta_{m+d-1}^*, \alpha_0^*, \ldots, \alpha_{n-1}^*)' \qquad .$$

Denote by $\beta(z^{-1})$ and $\alpha(z^{-1})$ the polynomials in z^{-1} with coefficients β_i^*, α_i^*, respectively. It can be seen from Figure 1.2 that the characteristic polynomial of the tuned closed-loop system is

$$P(z^{-1}) = A(z^{-1})\beta(z^{-1}) + z^{-1}B(z^{-1})\alpha(z^{-1}) \qquad .$$

Defining $V(z^{-1})$ as

$$V(z^{-1}) = (A(z^{-1}), \ldots, z^{-m-d+1} A(z^{-1}), z^{-1}B(z^{-1}), \ldots, z^{-n}B(z^{-1}))' \quad ,$$

the characteristic polynomial can be rewritten as

$$P(z^{-1}) = V(z^{-1})' \, \Theta^{*} \quad .$$

With this expression for the characteristic polynomial, some properties of the tuning condition can be stated.

<u>Property A1</u>: If

$$B(z_i^{-1}) \neq 0 \quad , \quad \forall i \in E \quad .$$

Then the tuning condition is equivalent to the following linear system:

$$z_i^{d-1} \, B(z_i^{-1}) = V(z_i^{-1})' \, \Theta^{*} \quad , \quad \forall i \in E \quad .$$

<u>Corollary A1</u>: Under the same assumption the tuning condition is satisfied iff

$$(B(z_i^{-1}))'_{i \in E} \in \text{Range } (V(z_i^{-1})')_{i \in E} \quad .$$

Corollary A1 shows that the tuning condition is satisfied generically if

$$n_E \leq m + d + n \quad .$$

As a consequence, $y_m^{*}(k)$ can contain no more than $\frac{m + d + n}{2}$ frequencies.

<u>Proof</u>: From (2.7), the tuning condition can be written as

$$y^{*}(k) = \Theta^{*'} \phi^{*}(k-d) \quad , \quad \forall k \quad ,$$

and using (1.2), the control is obtained from

$$y_m^{*}(k+d) = \Theta^{*'} \phi^{*}(k) \quad , \quad \forall k \quad .$$

Since $\Theta^{*'}$ is a constant, it follows readily that

$$y^{*}(k) = y_m^{*}(k) \quad , \quad \forall k \quad .$$

Now let $T_y(z^{-1})$ be the transfer function between $y_m^*(k+d)$ and $y^*(k)$. From the state space representation, we have:

$$T_y(z^{-1}) = h'(zI - (F - G_1 \frac{\theta_r^{*\prime}}{\beta^*} J))^{-1} \frac{G_1}{\beta^*}$$

And from a classical computation,

$$T_y(z^{-1}) = \frac{z^{-1}B(z^{-1})}{P(z^{-1})} \quad .$$

The identity of the sequences $\{y^*(k)\}$ and $\{y_m^*(k)\}$ yields

$$\sum_{i \in E} y_{mi}^*(z_i^d T_y(z_i^{-1}) - 1)z_i^k = 0 \quad , \quad \forall k \quad .$$

But since the sequences $\{z_i^k\}$, $i \in E$ are linearly independent, this relation shows that the tuning condition implies

$$z_i^d T_y(z_i^{-1}) - 1 = 0 \quad , \quad \forall i \in E \quad ,$$

or

$$\frac{z_i^{d-1}B(z_i^{-1})}{V(z_i^{-1})'\theta^*} - 1 = 0 \quad , \quad \forall i \in E \quad .$$

But we know that $B(z_i^{-1})$ is not zero, therefore, $V(z_i^{-1})'\theta^*$ is not zero. We have obtained the following necessary condition:

$$z_i^{d-1}B(z_i^{-1}) = V(z_i^{-1})'\theta^* \quad , \quad \forall i \in E \quad .$$

Clearly, this is also a sufficient condition. □

Corollary A2: Under Assumption A1,

$$\{u^*(k-d)\} = T_y(q^{-1})\ \{u^*(k)\} \quad , \quad \{y^*(k-d)\} = T_y(q^{-1})\ \{y^*(k)\} \quad .$$

Proof: The second relation has already been established. For the first one, we notice that $u^*(k)$ is the N-periodic output of an exponentially stable linear system (thanks to Assumption A1) with $y_m^*(k)$ as input. Therefore, $u^*(k)$ can also be written as

$$u^*(k) = \sum_{i\in E} u_i\, z_i^k \quad , \quad \forall k \quad .$$

The conclusion follows from the identity

$$T_y(z_i^{-1}) = z_i^{-d} \quad , \quad \forall i \in E \quad .$$

Property A2: If the tuning condition is satisfied and

$$B(z_i^{-1}) \neq 0 \quad , \quad \forall i \in E \quad .$$

Then uniqueness of Θ^* and persistence of excitation (Assumption A3) are equivalent properties.

Proof: Let $T_u(z^{-1})$ be the transfer function between $y_m^*(k+d)$ and $u^*(k)$, we have

$$T_u(z^{-1}) = \frac{A(z^{-1})}{P(z^{-1})} \quad .$$

Let $T_\phi(z^{-1})$ be the transfer function between $y_m^*(k+d)$ and $\phi^*(k)$. From the state representation, we have

$$T_\phi(z^{-1}) = z^d\ H(zI - (F - G_1 \frac{\Theta_r^{*\prime}}{\beta^*} J))^{-1} \frac{G_1}{\beta^*} \quad .$$

But also comparing the definitions of $\phi^*(k)$ and $v(z^{-1})$ and using the expression of $T_u(z^{-1})$, $T_y(z^{-1})$

$$T_\phi(z^{-1}) = \frac{V(z^{-1})}{P(z^{-1})} \quad .$$

This expression allows us to rewrite the persistence of excitation condition in the frequency domain as

$$\sum_{k=0}^{N-1} \phi^*(k)\phi^*(k)' = \sum_{i\in E} |y^*_{mi}|^2 \frac{\overline{V(z_i^{-1})}\, V(z_i^{-1})'}{|P(z_i^{-1})|^2} \quad .$$

Both y^*_{mi} and $P(z_i^{-1})$ are not equal to zero for $i \in E$, therefore, the fact that this matrix is positive definite is equivalent to the fact that the matrix $(v(z_i^{-1})'_{i\in E})$ is full rank. But with Property A1 we know that this is equivalent to the uniqueness of Θ^*.

Let us now prove Property A1.

<u>Lemma</u>: For any Θ^*, we have the following identity:

$$\Theta^{*\prime} H(zI - (F - G_1 \frac{\Theta_r^{*\prime}}{\beta^*} J))^{-1} \frac{G_1}{\beta^*} = z^{-d} \quad .$$

<u>Proof</u>: This transfer function is just $z^{-d}\Theta^{*\prime} T_\phi(z^{-1})$ where $T_\phi(z^{-1})$ is defined in the proof of Property A2. Since we have

$$T_\phi(z^{-1}) = \frac{V(z^{-1})}{V(z^{-1})'\Theta^*} \quad ,$$

the conclusion follows readily.

Now to prove Property A1, we have to show that the transfer function

$$H(z^{-1}) = z^{-d} + (h' - \Theta_*' H)(zI - (F - G_1 \frac{\Theta_r^{*\prime}}{\beta^*} J))^{-1} \frac{G_1}{\beta^*}$$

with $\{\phi^*(k)\}$ as input, has $\{\phi^*(k-d)\}$ as an output. From this lemma, we can rewrite $H(z^{-1})$ simply as

$$H(z^{-1}) = h'(zI - (F - G_1 \frac{\Theta_r^*}{\beta^*} J))^{-1} \frac{G_1}{\beta^*} \quad .$$

But this is just $T_y(z^{-1})$. Therefore, the conclusion follows from the definition of $\phi^*(k)$ and Corollary A2.

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